Relationships Between Nodes, Connections and Faces on a Planar Graph

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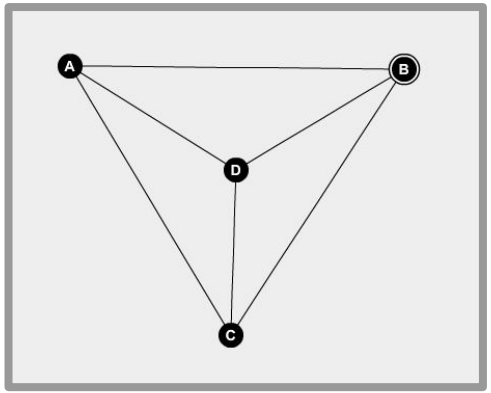
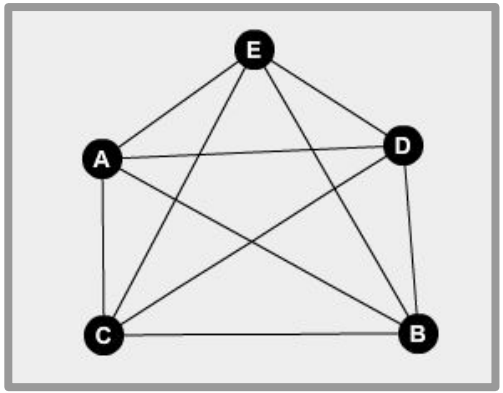
Introduction

- What is a Graph?
  - graph - a group of nodes joined together by lines
  - planar graph - an arrangement of a graph where the lines only intersect at the nodes



Left: Planar Graph



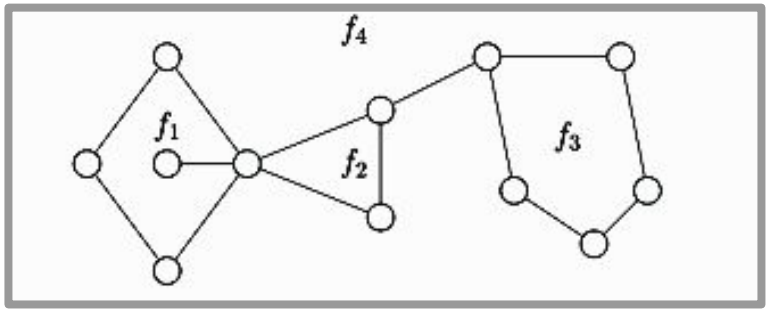
Right: Non-Planar Graph

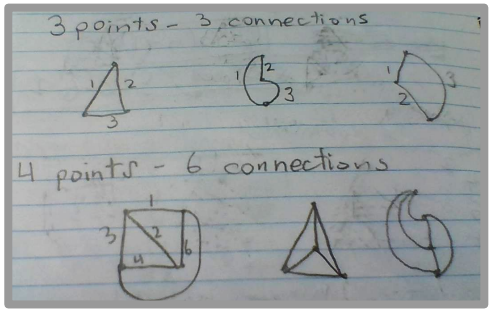
Problem Statement

- Is there a relationship between the number of possible connections, number of faces, and number of nodes in a maximum connection planar graph and does the arrangement of the nodes matter? Why?

Results

- Relationships
  - Connections & Nodes - $C = 3(n-2)$
  - Relationship of Faces - $f = c - (n-2)$ or $f - c + n = 2$
  - Note : a face on a planar graph is the enclosed space between the lines, but the space outside of the graph is counted as a single face.



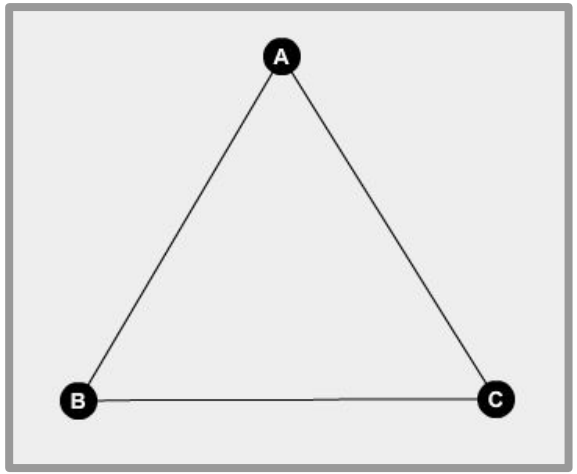


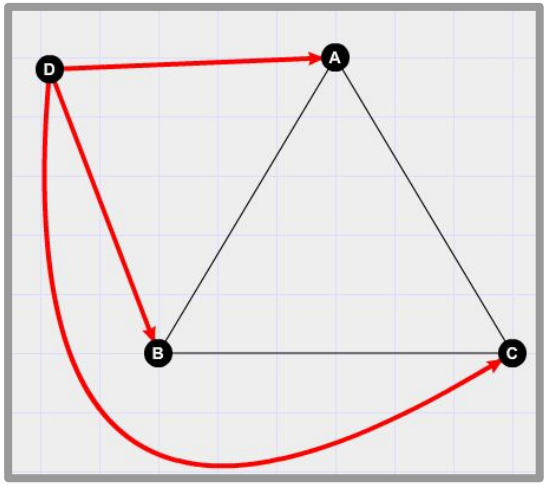
# of nodes	# of faces	# of connections
3	2	3
4	4	6
5	6	9
6	8	12
7	10	15
8	12	18
9	14	21
10	16	24
11	18	27
12	20	30
13	22	33
14	24	36

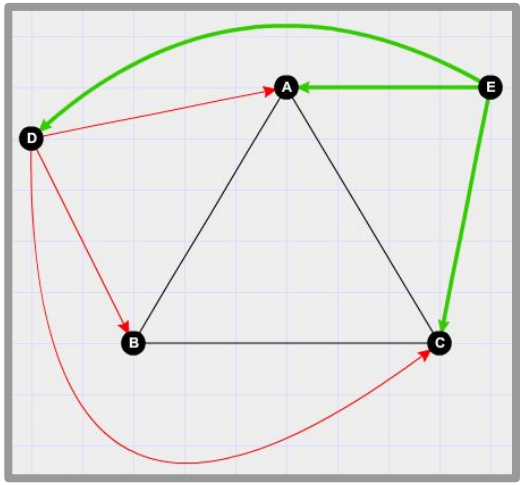
Results - continued

$$C = 3(n-2)$$

Why is that for every one node added, three more connections are added?



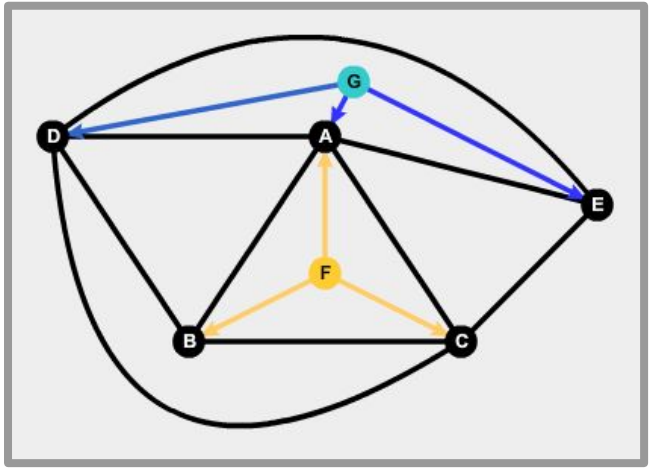


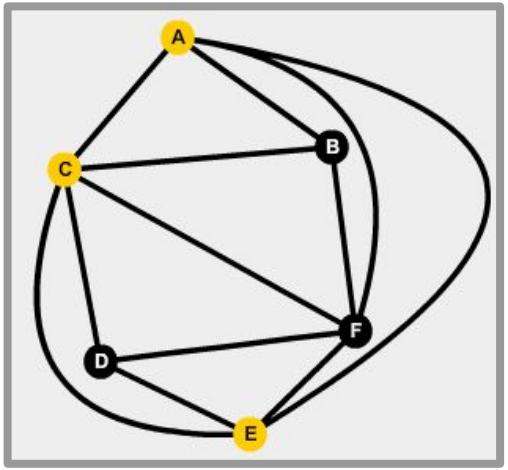


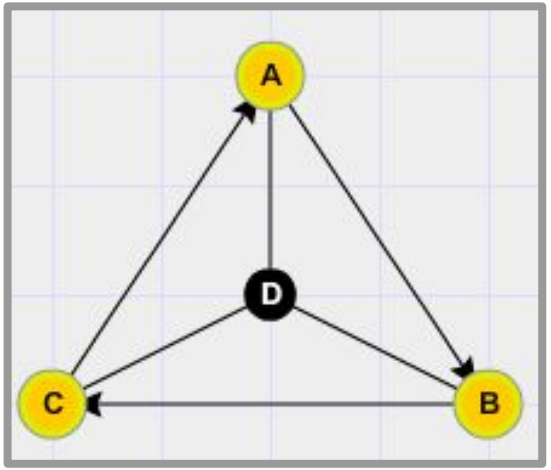
Left: Triangle ABC **Center**: Node D is added **Right**: Node E is added

Results - continued

- Outermost Nodes and Adding Points inside of a face
  - Every arrangement of any amount of nodes will only have three outermost points
  - Every max planar graph face is enclosed by only three nodes







Conclusion

- There *is* a relationship between the number of connections, faces, and nodes :

$$C = 3(n-2) \qquad f = c-(n-2) \quad \text{or} \quad f-c+n=2$$

- The arrangement of the nodes in a planar graph does not matter.
- What I Would Do Next:
  - Find reasoning behind the relationship of faces equation

Questions are welcome!