Planar Graphs

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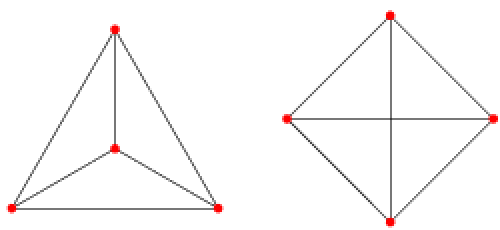
Introduction

A *planar graph* is a connected graph that can be drawn without any connections crossing. These kinds of graphs can appear to be non-planar, but can be redrawn in a way where none of the connections cross. This is called the planar representation of a graph.



A non-planar representation of a planar graph can be re-assembled to show the planar representation

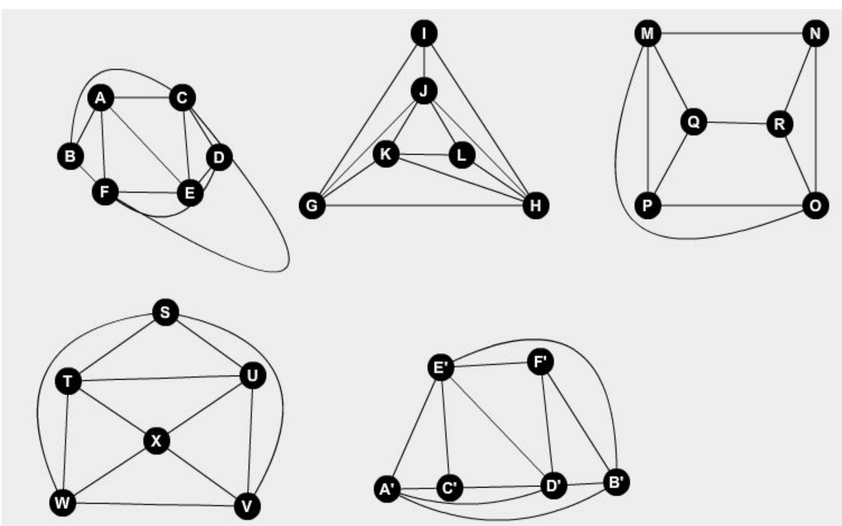
A planar graph's connections and vertices divide the plane into regions known as faces. Faces can only be counted when the graph is drawn in the planar representation form. Each divided section of the graph is a face, and the "outside" region is also considered a face.

Problem Statement

For my problem, I wanted to see if the shape of a planar graph would affect the maximum number of connections you could have. I also wanted to know if there was a relationship between the number of vertices, edges and faces.

Results

First, I drew graphs of different shapes, but using the same number of vertices. I started with 6 vertices, using 6 different shapes: Hexagon, triangle, square, pentagon, trapezoid, and a line. For all the shapes that would normally need less than 6 vertices, I put the extra ones in the middle. When I finished the shapes with 6 vertices, I moved on to 8, then 12, and finally 15.



When I finished creating planar graphs with 15 vertices, I realized that the shape of a planar graph wouldn't make a difference in the maximum number of edges that were possible because they all had the same number of vertices.

For the second part of the problem, I drew a table that included the number of faces and edges you had when you have a certain number of vertices. I used point-slope form to find the relationship between the number of vertices and edges, vertices and faces, and edges and faces.

Vertices	Edges	Faces
1	0	0
2	1	1
3	3	2
4	6	4
5	9	6
6	12	8
7	15	10
8	18	12
9	21	14
10	24	16

I discovered that the relationship between the number of vertices and edges was $y=3x-6$, where y is the maximum number of edges, and x is the number of vertices. I also discovered that the number of vertices and faces were also related, and the equation I found was $y=2x-4$, where y is the number of faces and x is the number of vertices. Another equation I found using point-slope form was $y=2/3x$, where y is the number of faces and x is the number of edges.

Finally, the last equation I wanted to find was the relationship between all three variables. I couldn't use point-slope form for this equation, because it only works for two variables. So I started by looking at the variables and trying to find a relationship between all three. I continued to test possible equations until I found $x-y+z=2$. x is the number of vertices, y is the number of edges and z is the number of faces.

Conclusion

In the end, I found that there was no relationship between the shape of a planar graph and the maximum number of edges you could have on that graph, however, I did find that there was a relationship between the number of vertices and the maximum number of edges and faces.

To make my problem harder, I would have tried to see if the same equations that I found would work for non-planar graphs, and why each time you add a vertex, you always get exactly 3 more edges.