Hodgkin-Huxley Network with STDP Learning - An Overview
Justin Skycak

---

1. Set Network Topology and Learning Parameters

Collect information necessary to implement network topology parameters (number N of neurons, initial adjacency matrix $W(0) = [w_{ij}(0)]$), STDP parameters (max fraction change A_+, A_- in weights, window $[\tau_+, \tau_-]$ of learning), and stimulus waveform $s(t)$. The GUI should use the parameters to give an estimate on runtime, and to construct the stimulus waveform, the user should input a mathematical function or draw (click and drag mouse) the waveform on a blank graph.

---

2. Collect constants

The average conductances and reversal potentials for sodium ($\overline{g}_{Na}, V_{Na}$), potassium ($\overline{g}_K, V_K$), and leak ($\overline{g}_L, V_L$) channels, along with the membrane capacitance C are found in [1].

---

3. Simulation

—–3.1 A Brief Overview of a Hodgkin-Huxley Network—–

Let $V_i(t)$ be the membrane potential $I_i(t)$ the current output of neuron i evolve over time.
If we know $I_i(t)$ and $V_i(0)$, then we can compute $V_i(t)$:

$$I_i(t) = C\frac{dV_i(t)}{dt} \implies V_i(t) = V_i(0) + \int_0^t \frac{I_i(t)}{C}dt$$

$V_i(0)$ can be taken as resting potential, so let us commence with finding $I_i(t)$. We can break $I_i(t)$ into an internal current $I_{int,i}(t)$ (due to electrode stimulus and internal ion channels) and an external current $I_{ext,i}(t)$ (due to activity in neurons to which it is connected). Hence,

$$I_i(t) = I_{int,i}(t) + I_{ext,i}(t)$$

where

$$I_{int,i} = s(t) - I_{Na,i}(t) - I_{K,i}(t) - I_{L,i}(t) \quad , \quad I_{ext,i} = \textstyle\sum_{j=1}^N w_{ij} I_j(t)$$

Hodgkin and Huxley's equations [1] can be rearranged to tell us how to compute the terms $I_{Na,i}(t)$, $I_{K,i}(t)$, and $I_{L,i}(t)$ in $I_{int,i}(t)$:

$$I_{Na,i}(t) = g_{Na}(V_i(t) - V_{Na}) \quad , \quad I_{K,i}(t) = g_K(V_i(t) - V_K) \quad , \quad I_{L,i}(t) = g_L(V_i(t) - V_L)$$

where

$$g_{Na} = \overline{g}_{Na} m_i(t)^3 h_i(t) \quad , \quad g_K = \overline{g}_K n_i(t)^4 \quad , \quad g_L = \overline{g}_L$$

are the conductances whose terms are governed by

$$\tfrac{dn_i(t)}{dt} = \tfrac{\alpha_{n,i}(t)}{\alpha_{n,i}(t)+\beta_{n,i}(t)} \quad , \quad \tfrac{dm_i(t)}{dt} = \tfrac{\alpha_{m,i}(t)}{\alpha_{m,i}(t)+\beta_{m,i}(t)} \quad , \quad \tfrac{dh_i(t)}{dt} = \tfrac{\alpha_{h,i}(t)}{\alpha_{h,i}(t)+\beta_{h,i}(t)}$$

where

$$\alpha_{n,i}(t) = 0.1 * \frac{10 - V_i(t)}{\exp\{\frac{10-V_i(t)}{10}\}-1} \quad , \quad \alpha_{m,i}(t) = 0.1 * \frac{25 - V_i(t)}{\exp\{\frac{25-V_i(t)}{10}\}-1} \quad , \quad \alpha_{h,i}(t) = 0.07 * \exp\{\frac{-V_i(t)}{20}\}$$

$$\beta_{n,i}(t) = .125 * \exp\{\frac{-V_i(t)}{80}\} \quad , \quad \beta_{m,i}(t) = 4 * \exp\{\frac{-V_i(t)}{18}\} \quad , \quad \beta_{h,i}(t) = \frac{1}{\exp\{\frac{30-V_i(t)}{10}\}+1}$$

—–3.2 Implementation of the Hodgkin-Huxley Network—–

We implement this system using first-order forward Euler approximation. First, we set each $V_i(0)$ to resting potential and use it to compute $\alpha_{n,i}(0)$, $\alpha_{m,i}(0)$, $\alpha_{h,i}(0)$, $\beta_{n,i}(0)$, $\beta_{m,i}(0)$, $\beta_{h,i}(0)$. Then we approximate $n_i(0) \approx \frac{dn_i(0)}{dt}$, $m_i(0) \approx \frac{dm_i(0)}{dt}$, $h_i(0) \approx \frac{h_i(0)}{dt}$. We then compute $g_{Na}(0)$, $g_K(0)$, $g_L(0)$, which we use to compute $I_{Na,i}(0)$, $I_{K,i}(0)$, $I_{L,i}(0)$, which we use to compute $I_{int,i}(0)$. We use the assumption that $I_{ext,i}(0) = 0$ to calculate $I_i(0)$.

Recursively, then, for $t = 0, \Delta t, 2\Delta t, ...$, we approximate the voltage by

$$\begin{aligned} V_i(t + \Delta t) &= V_i(t) + \frac{dV_i(t)}{dt}\Delta t \\ &= V_i(t) + \frac{I_i(t)}{C}\Delta t \end{aligned}$$

and we update

$$n_i(t + \Delta t) = n_i(t) + \frac{dn_i(t)}{dt}\Delta t \quad , \quad m_i(t + \Delta t) = m_i(t) + \frac{dm_i(t)}{dt}\Delta t \quad , \quad h_i(t + \Delta t) = h_i(t) + \frac{dh_i(t)}{dt}\Delta t.$$

We use these values to calculate $g_{Na}(t+\Delta t)$, $g_K(t+\Delta t)$, $g_L(t+\Delta t)$, which we use to compute $I_{Na,i}(t+\Delta t)$, $I_{K,i}(t + \Delta t)$, $I_{L,i}(t + \Delta t)$, which we use to compute $I_{int,i}(t + \Delta t)$. Lastly, we calculate $I_{ext,i}(t + \Delta t) = \sum_{j=1}^{N} w_{ij} I_j(t)$, and we update $I_i(t + \Delta t) = I_{int,i}(t + \Delta t) + I_{ext,i}(t + \Delta t)$.

—–3.3 A Brief Overview of STDP Learning—–

STDP learning describes how neural connectivity changes depend on relative timing of neural spikes [2]. Specifically, when presynaptic and postsynaptic spikes are induced in neurons, resulting in presynaptic spikes at times $\{t^a_{pre}\}_{a\geq 1}$ and postsynaptic spikes at times $\{t^b_{post}\}_{b\geq 1}$, it is found [3] that the relative change in connection weight is given by

$$\frac{\Delta w_{post,pre}}{w_{post,pre}} = \sum_{a,b} \mathcal{W}(t^b_{post} - t^a_{pre})$$

where $\mathcal{W}$ is an STDP function. A choice of

$$\mathcal{W}(t) = \begin{cases} A_+ \exp\{-\frac{t}{\tau_+}\} & t > 0 \\ A_- \exp\{-\frac{t}{\tau_-}\} & t < 0 \end{cases}$$

with time constants $\tau_+, \tau_- \approx 10$ ms has been consistent empirical findings [4].

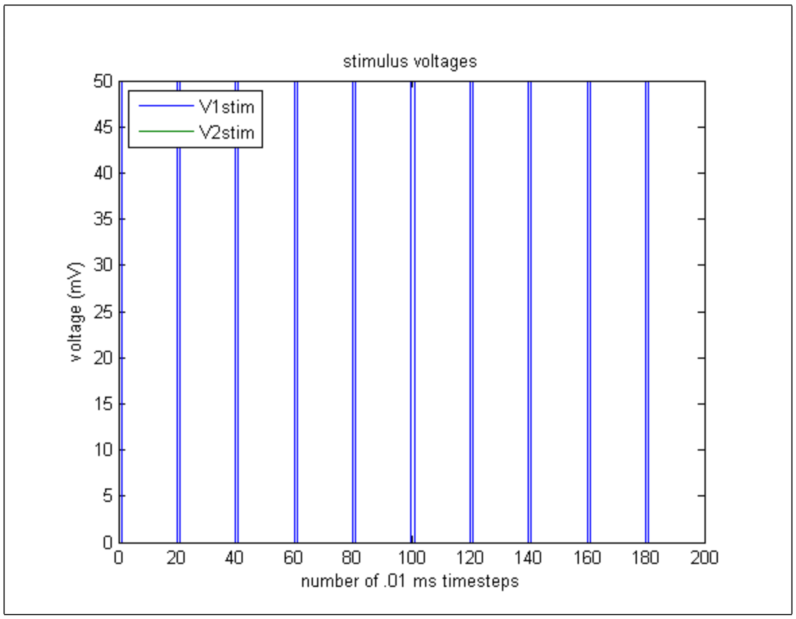
Whenever $V_i(t)$ is updated, we check whether it is increasing and has crossed a threshold voltage $\phi = 60mV$ which we take to imply an action potential has occurred. If $V_i(t) < \phi < V_i(t + \Delta t)$, then we take $t + \Delta t$ to be a crossover time for neuron i.

For simplicity, we begin updating weights only after an amount of time equal to our update window ℓ has passed, and we stop updating weights ℓ steps before the end of simulation. After updating all the neurons in a given timestep, we look at the crossover times to determine the time differences in action potentials, and we use $\mathcal{W}(t)$ to adjust the weights accordingly.

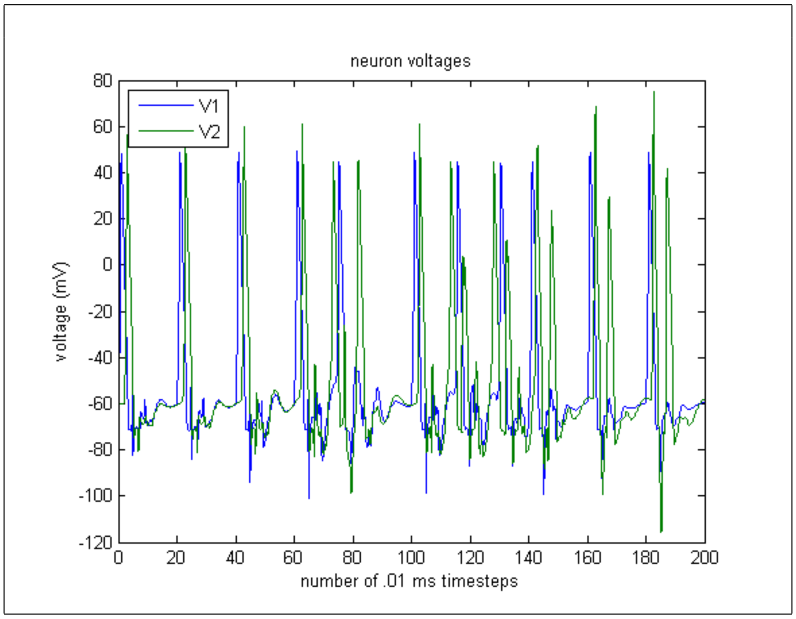
---

4. Example and Interpretation of a 2-Neuron Simulation

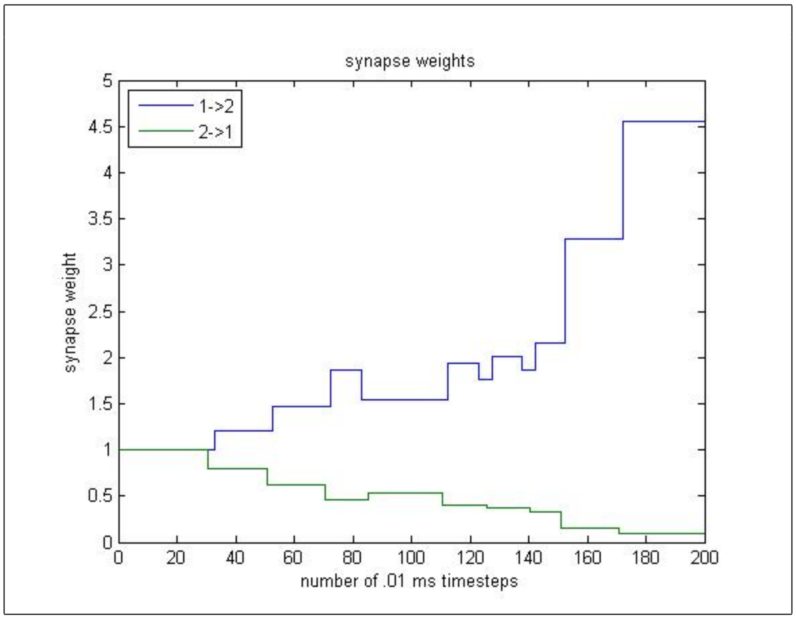
Here, we see an example trial. Neuron 1 is stimulated with a periodic 50 mV stimulus, as shown below:



Assuming that the initial $1 \rightarrow 2$ and $2 \rightarrow 1$ connection weights are both 1, we observe the following activity:



Using the STDP scheme, the $1 \rightarrow 2$ and $2 \rightarrow 1$ synapse weights change as follows:



—–Explanation of Steps 0-40—–

The weights remain constant because the learning interval has not yet been reached.

—–Explanation of Steps 40-80—–

We see that for steps ~ 40 to ~ 80, the weight of the $1 \rightarrow 2$ synapse increases. This makes sense because neuron 1 is being stimulated, neuron 1 excites neuron 2 with weight 2, and so stimulation in neuron 1 causes neuron 2 to spike quickly afterwards.

On the other hand, the weight of the $2 \rightarrow 1$ synapse is decreasing. Although spikes in neuron 2 excite neuron 1, they occur right after neuron 1 has spiked, so neuron 1 is in its refractory period and cannot spike. Thus, the $2 \rightarrow 1$ synapse weight decreases.

—–Explanation of Steps 80-140—–

During this period, the change in weight causes neuron 2 to become very eager to spike whenever V_1 increases sharply, and the $1 \to 2$ weight oscillates. We see that between steps $\sim$ 80 and $\sim$ 140, a sharp increase in V_1 causes neuron 2 to spike. However, the sharp increase in V_1 does not lead into a spike in neuron 1, so the $1 \to 2$ weight decreases. However, neuron 2's spike incites a spike in neuron 1, which incites another spike (of lesser magnitude) in neuron 2, so the $1 \to 2$ weight increases. This explains the $1 \to 2$ oscillation.

When neuron 2 spikes immediately prior to neuron 1, the $2 \to 1$ synapse increases. However, when neuron 2 spikes in response to a spike from neuron 1, the $2 \to 1$ synapse decreases. Overall, the $2 \to 1$ synapse remains roughly constant during this interval because its magnitude is so low.

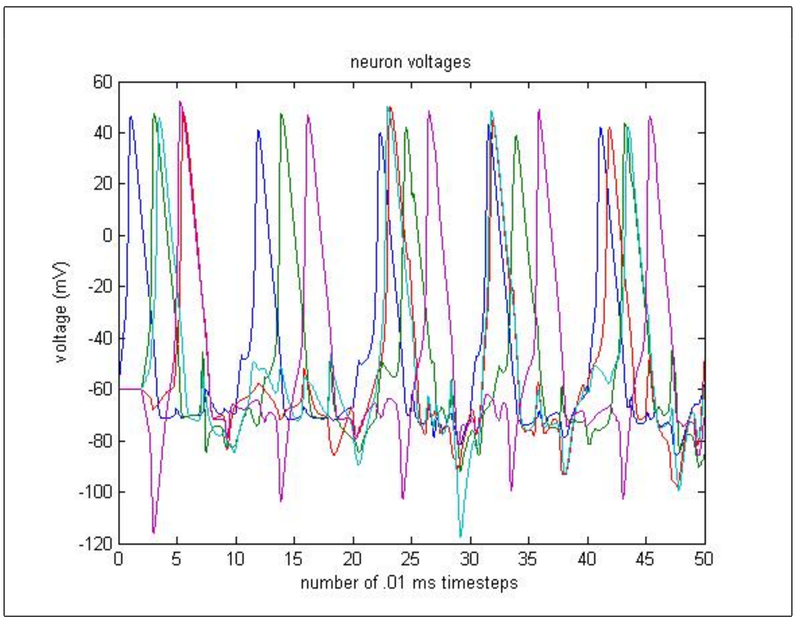
—–Explanation of Steps 130-200—–

At these steps, the $1 \to 2$ synapse weight is so large that when neuron 1 spikes, it incites a spike in neuron 2, and when neuron 1 returns from hyperpolarization to resting state, neuron 2 spikes again. Because each spike in neuron 1 is followed by two spikes in neuron 2, the $1 \to 2$ weight increases greatly.

However, spikes in neuron 1 do not immediately follow spikes in neuron 2, so the $2 \to 1$ synapse weight becomes closer and closer to 0.

---

4. A 5-Neuron Simulation



References

[1] Hodgkin, A. L., & Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. The Journal of Physiology, 117(4), 500544.

[2] Jesper Sjstrm and Wulfram Gerstner (2010), Scholarpedia, 5(2):1362.

[3] Gerstner, W., Kempter R., van Hemmen J.L., and Wagner H. (1996). A neuronal learning rule for submillisecond temporal coding. Nature, 386:76-78.

[4] Zhang, L. I., Tao, H. W., Holt, C. E., Harris, W. A., and Poo, M.-M. (1998). A critical window for cooperation and competition among developing retinotectal synapses. Nature, 395:37-44.