

HBar Math Circle @ Walden
Class 1 [2.20.2018]

[Problem 1.1]

Exactly one of the Dolphins is telling the truth.

- Dolphin 1: "Exactly one of us is lying!"
- Dolphin 2: "Exactly two of us are lying!"
- Dolphin 3: "Exactly three of us are lying!"
- Dolphin 4: "Exactly four of us are lying!"
- Dolphin 5: "Exactly five of us are lying!"
- Dolphin 6: "Exactly six of us are lying!"
- Dolphin 7: "Exactly seven of us are lying!"

Which dolphin is telling the truth?

[Problem 1.2]

A pizza is listed on the menu for \$10, but there is also a 9% tax, a 10% delivery, and a 15% gratuity. How much do you actually pay?

[Problem 1.3]

Bob's test has 25 questions, each with only two answer choices.

- correct answer = +1 point
- incorrect answer = +0 points

How many ways can the questions be answered such that it has a score of 24? What about a score of 23?

[Problem 1.4]

In how many ways can we paint these rectangles red, yellow, or blue such that each rectangle cannot be horizontally or vertically adjacent to a rectangle with the same color?



[Problem 1.5]

Bob got his test back. He had topped the class with a score of 23. The lowest person scored a 19. The other two people scored a 21. What was the median of the median, the mean, and the mode?

[Problem 1.6]

In the jungle, currency consists of bananas, mangos, and kiwi.

3 bananas = 1 mango

5 kiwis = 1 mango

If a gorilla goes to a fruit shop with have 5 bananas and 3 kiwis, how many mangoes can he buy?

[Problem 1.7]

What is

$$(100 - 10)(90 - 9)(80 - 8) \dots (-80 + 8)(-90 + 9)(-100 + 10)?$$

[Problem 1.8]

A 3x3x3 Rubik's Cube is sitting on the table. I pour water all over it, without flipping it over. What fraction of the 27 smaller cubes have exactly 2 wet sides?

[Problem 1.9]

5 kids got in a line in a random order for pictures. If the teacher calls on two kids standing next to each other, they switch spots in the line.

The teacher wants them ordered by height, shortest in the front, tallest in the back. Is it possible to do this just by calling on them? How do you know?

[Problem 1.10]

In the previous problem, what is the maximum number of times the teacher could have to call on a pair of students, to order them all?

[Problem 1.11]

In the previous problem, if there were 100 students?

[Problem 1.12]

Use four 5s and some of the symbols $+$, \times , $-$, and \div to give expressions for 0, 1, 2, and 5

[Problem 1.13]

A leprechaun hands you 12 gold coins and a balance scale. He says:

*“One of the coins is fake!
It is lighter than the rest.
You may use the scale only 3 times!
Find the coin, the rest you win!”*

How can you find the fake coin?

[Problem 1.14]

Two mountain villages are connected by a road. Not a single stretch of this road is flat: the road always goes either uphill or downhill. A round trip on a bus between these two villages takes 6 hours. The speed of the bus on any uphill section of the road is 30 miles per hour, and on any downhill section it's 60 miles per hour. Find the distance between these villages

[Problem 1.15]

Mr. Spider has 8 legs and 2 socks. How many ways can he wear his 2 socks?

[Problem 1.16]

In the previous problem, what if Mr. Spider has 3 socks?

[Problem 1.17]

Mr. Spider wants to select 3 socks from his drawer, which contains 7 socks in total: red, orange, yellow, green, blue, indigo, and violet. How many different color combinations can he choose?

[Problem 1.18]

Haley has a fair eight-sided die that has a different number from 1 to 8 on each side. If she rolls this die twice, what is the probability that the second number rolled is greater than or equal to the first number?

[Problem 1.19]

What is the greatest number that evenly divides the sum of any six consecutive whole numbers?

[Problem 1.20]

Joey goes to the beach, and enters a contest: each person is given an 8-gallon jug and a 5-gallon jug, which they may refill and empty at will (both into the ocean and into each other). The first person to have exactly one gallon in a jug wins. How can Gerald win?