Posing a Problem

Research and Presentation in Mathematics | Justin Skycak

Class Outline. We will begin by posing problems in a variety of problem domains. Then, you will choose a research problem to work on for two weeks. Your problem may be one of those posed in class, or an original problem of your own. In the last two weeks, you will write a final report and give a brief talk to the class.

Posing a Problem. To pose a research problem, we first ask a question. That initial question may be too easy or too difficult to solve, so we keep modifying the question until we reach a problem that is interesting yet approachable.

Practice. Below are examples of problems in various fields of mathematics. During class, we will attempt to pose as many problems as we can in these fields, so that you have many ideas in mind when choosing your problem.

- *Probability.* What is the probability that the triangle formed by the origin and two points randomly chosen the unit circle has area greater than ½?

- *Recursion.* What pattern does a ball make if it is thrown with angle θ within the unit square?

- *Optimization.* A business is open 7 days/week, but employees can only work 5 days/week, and each one has particular days that they want off. What is a schedule such that on average everyone works the same number of days they don't want to, and such that that number is minimized? (The schedule does not have to be weekly regular.)

- *Logic.* What would be the mathematical consequences of allowing exponents to distribute over sums, or allowing fractions to be added straight across (like they can be multiplied)?

- *Visualization/Experiment.* Why does the graph of $\sin(y) = \cos(x)$ look the way it does? What about $\sin(y) = \cos(x)\tan(y)$, or $\sin(y)\tan(x) = \cos(x)\tan(y)$?

- *Graph Theory.* What are necessary and/or sufficient conditions for a graph to be (or not be) planar?

- *Algorithms*. What algorithm will allow a robot to autonomously navigate a partially blocked grid, if it has no vision capabilities and can only sense when it runs into something? (Imagine you were blindfolded -- how would you do it?)

- *Modeling.* Create and analyze a mathematical model for a real-life process, such as traffic flow. Ensure that your model replicates real-life phenomena, such as traffic jams.

Choosing your Problem. When choosing a problem, you should strike a balance between your interests and your strengths. For example, if you think visualizing/experimenting with graphs of trig functions sounds fun, but you don't have a good understanding of trig functions, then you will have trouble making research progress. Likewise, if you have a good understanding of trig functions, but you aren't excited by the idea of explaining a graph, then it will be difficult to excite your audience with a compelling research talk.